Chapter 10

Evaluating Molecular Statements

In this chapter, we look at how you figure out the truth values of molecular statements. This will involve review of and reflection upon the meanings of the three logical operators we've learned so far.

Evaluating statements

To ***evaluate*** a statement is, very simply, to ***determine its truth value***. (The word 'evaluate' comes from a combination of Latin words meaning 'to bring the value out'.) Evaluating statements is thus something you do all the time.

We often have to evaluate atomic statements. Suppose, for instance, that you have parked your convertible outside your school with its top open, and have gone inside, when someone comes in and announces that it's raining. There are two possibilities. His statement that it's raining might be true, in which case you need to run outside and close the top on your convertible, or it might be false, in which case he's probably trying to get you to look all worried and run outside to close the top on your convertible. (This example is drawn from an actual occurrence at a meeting of our school board. The practical joke didn't succeed, as the owner of the convertible didn't believe the statement that it was raining.)

It's not only for avoiding practical jokes that we must evaluate statements, however; there are more serious reasons for doing so. Consider a statement like this one, uttered by someone who has called the police:

Someone is trying to kill me. (1)

The police must decide whether this statement is true, or whether there's a credible chance that it might be true. If it is true, or might be true, then they will need to investigate immediately and determine how to protect the person whose life is in danger. If it's false (perhaps the person who has called them is insane), then they should not waste any of their time on it. What makes this example so urgent is that if the police don't guess correctly, then someone might lose his life.

And of course there are situations in which we must evaluate atomic statements whose seriousness is between that of the two examples I've given—more serious than a practical joke, but less serious than a police emergency call. Most everyday examples of statement evaluation fall in this broad range. Consider the following atomic statements, and try to imagine a situation in which someone might need to evaluate each of them:

Grand Onion has the best price on yogurt right now. (2)

It's going to snow this evening. (3)

School's opening is delayed two hours today. (4)

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Uncooked rhubarb is poisonous. (5)

Snears sells the best tools. (6)

I bought this car new. (7)

Parsnip is a very well-behaved cat. (8)

Logic is very, very easy. (9)

U.S. Treasury bonds are very safe investments. (10)

It's pretty easy to imagine situations in which someone might care about the truth value of each of these statements, isn't it? This suggests how frequently we find ourselves having to evaluate atomic statements every day.

What we want to explore in this chapter is how to evaluate molecular statements. To evaluate a molecular statement, you will generally need to evaluate all the atomic statements contained within it, so for the rest of this chapter, I'm going to restrict myself to atomic statements whose truth values can be established very easily. We want to focus on the truth values of the molecular statements we build out of our atomic statements, not on the atomic statements themselves.

Logical operators and truth values

So far, we've studied three logical operators: negation, conjunction, and disjunction. Let's look at each to determine how we would go about evaluating them.

We've already looked at this for negation, so we'll just review it now. Remember what the negation operator means, or what it's used for. It asserts that the statement being negated is false, as in this example:

It is not the case that 'K' is the symbol for the element potassium. (11)

To figure out whether the negation itself is true or false, we need to know whether the atomic statement negated by it is true or false. If the atomic statement is false, then the negation is true, since the negation says that the atomic statement is false. But if the atomic statement is true, then the negation is false, for the same reason.

So, is the atomic statement in (11) true or false? That's easily determined from a dictionary, for example. Looking it up (if necessary), we see that the atomic statement is true. That is, 'K' is in fact the symbol for potassium. What does that tell us about the truth value of (11) as a whole? It must be false, since it asserts that its atomic statement is false, which is wrong.

It can be useful to build a Parsnip tree for evaluating molecular statements. Here's one for statement (11):

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Negation
('it is not the case that')

'K' is the symbol for the
element potassium.

When using a Parsnip tree to evaluate such a statement, it is useful to write the truth value of each part right above it. Let's start by marking the truth value of the atomic statement:

Negation
('it is not the case that')

T
'K' is the symbol for the
element potassium.

Now whenever we have a negation of a true statement, we know that the negation is false (since it says that the statement that's being negated is false, which is it isn't). So we can write the truth value of the negation above it, like this:

F
Negation
('it is not the case that')

T
'K' is the symbol for the
element potassium.

Of course, if the statement being negated had been false, then the negation would be true, as in the following example.

T
Negation
('it is not the case that')

F
'P' is the symbol for the
element potassium.

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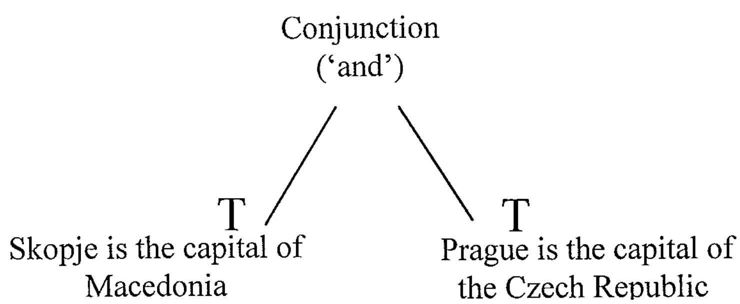
To state the truth value rule for negation very simply, ***the truth value of a negation is always the reverse of the truth value of its operand***.

Let's turn our attention to conjunction. Do you remember what conjunction is used for? It's used to assert that both conjuncts are true. So what are the truth conditions for the conjunction itself? Well, since it says that both conjuncts are true, then it will be true if the conjuncts are both true, and it will be false otherwise.

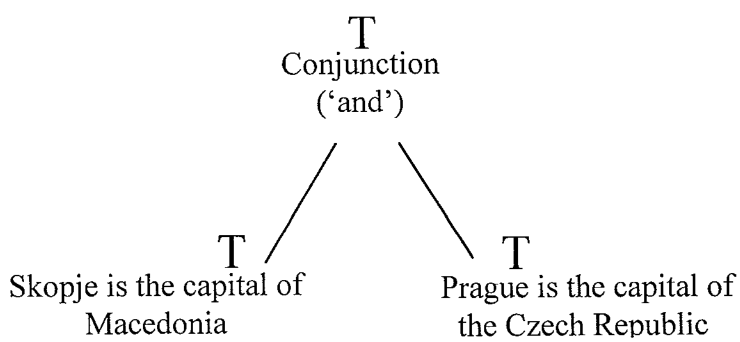
Here's an example of a conjunction of two true statements:

Skopje is the capital of Macedonia, and Prague is the capital of the Czech Republic. (12)

And here's the Parsnip tree for (12), with the truth values of its conjuncts labeled:



What is the truth value of the whole thing? The whole thing is a conjunction, and a conjunction asserts that both its conjuncts are true. Are both its conjuncts true? Yes. So the whole statement (that is, the whole conjunction) is true, and we can label it accordingly on our Parsnip tree:



What about conjunctions where one or both conjuncts are false? Such conjunctions can't be true, since they assert that both conjuncts are true, and they aren't. Let's illustrate these with 'empty' Parsnip trees, from which I've omitted the atomic statements (but not their truth values). There are three cases:

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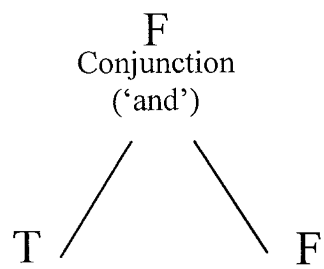
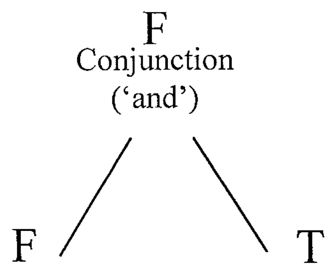
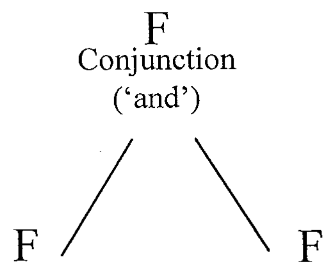


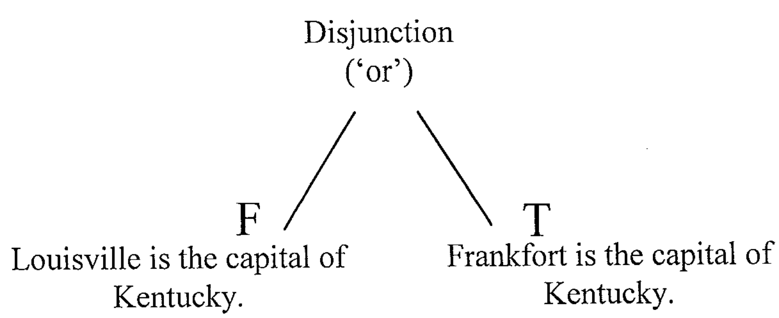




Think carefully about each of these cases, and make sure you understand why the conjunction is false in each instance. To summarize before we move on, we can say that ***a conjunction is true when (and only when) both of its conjuncts are true***.

What about disjunction? Recall its meaning. Disjunction—remember that we're talking about inclusive disjunction here—asserts that at least one of its disjuncts is true. So if both disjuncts are true, is the disjunction true? Yes, because at least one disjunct is true. What if one disjunct is true and the other is false? Then the disjunction is true, because at least one disjunct is true. In fact, the only case in which a disjunction is false is that in which both disjuncts are false, because in that case it isn't the case that at least one disjunct is true.

Let's illustrate one of the four cases with a Parsnip tree:



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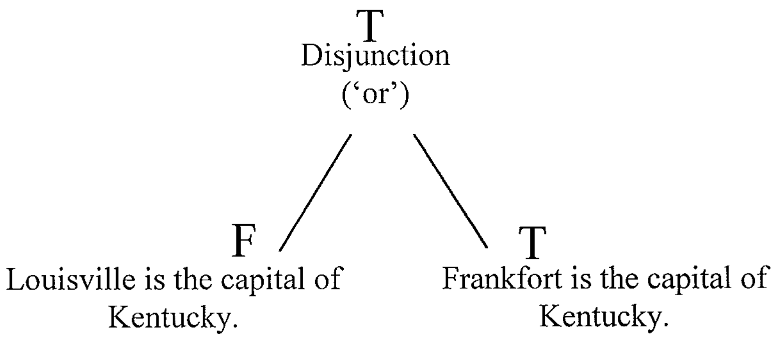
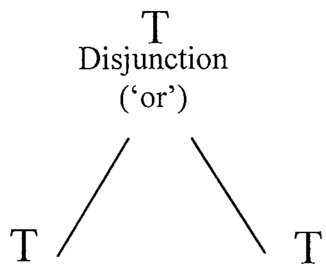
Given (as I'm sure you are aware) that Frankfort is and Louisville is not the capital of Kentucky, is the disjunction of the two statements true? Yes, it is. If someone uttered (13), you would have to say that he had spoken truly:

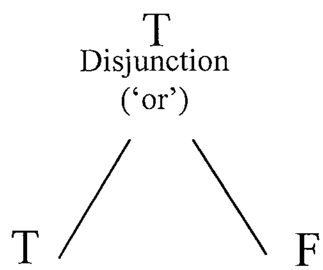
Louisville is the capital of Kentucky or Frankfort is the capital of Kentucky. (13)

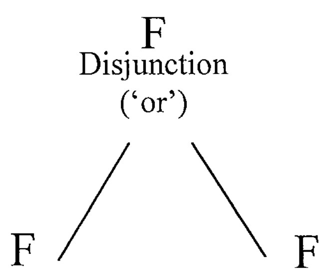
That's because the disjunction operator says only that at least one of the operands is true, which is true in this case. So we can mark our Parsnip tree with the truth value of the disjunction, thus:



Now let's summarize the other three possibilities for disjunction using empty Parsnip trees, as we did for conjunction:







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We can summarize the rules for the truth values of disjunctions by saying that ***a disjunction is true unless both its disjuncts are false***.

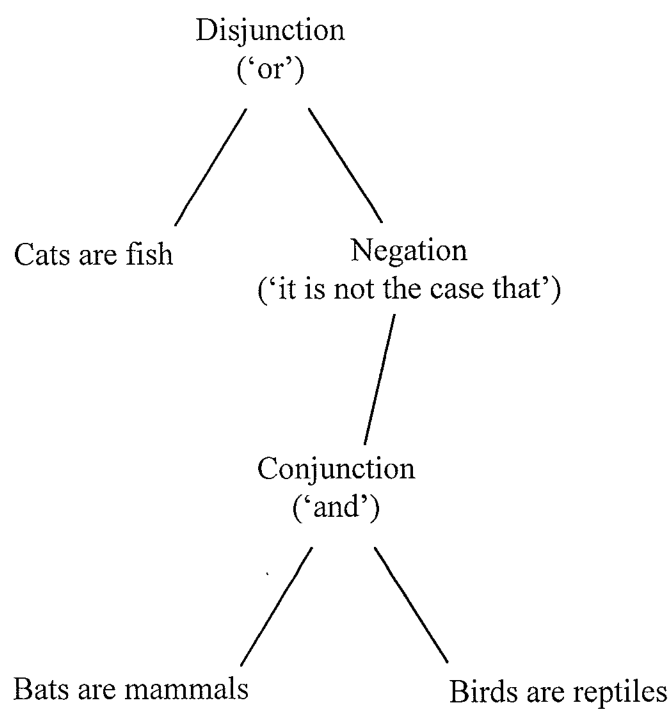
Evaluating complicated molecular statements

In the previous section, you were given Parsnip trees for each of the three logical operators, illustrating every possible combination of truth values for their operands. You can use these examples (whether they were empty Parsnip trees or not) to determine the truth values of any more complicated molecular statements you might meet. Let's consider a few examples. For the first one, I'll spell out the steps in considerable detail.

Here's a molecular statement for us to work with:

Cats are fish, or it is not the case that bats are mammals and that birds are reptiles. (14)

By now, I'm sure you know how to build the Parsnip tree for such a statement, so I'll just give it to you:



To evaluate this statement, using the Parsnip tree, we must ***first evaluate each atomic statement in it and write its truth value on the Parsnip tree***. In this example, there are three atomic statements. The first, 'Cats are fish', is false; the second, 'Bats are mammals', is true; and the third, 'Birds are reptiles', is false. Writing these truth values on the Parsnip tree, we get a tree that looks like this:

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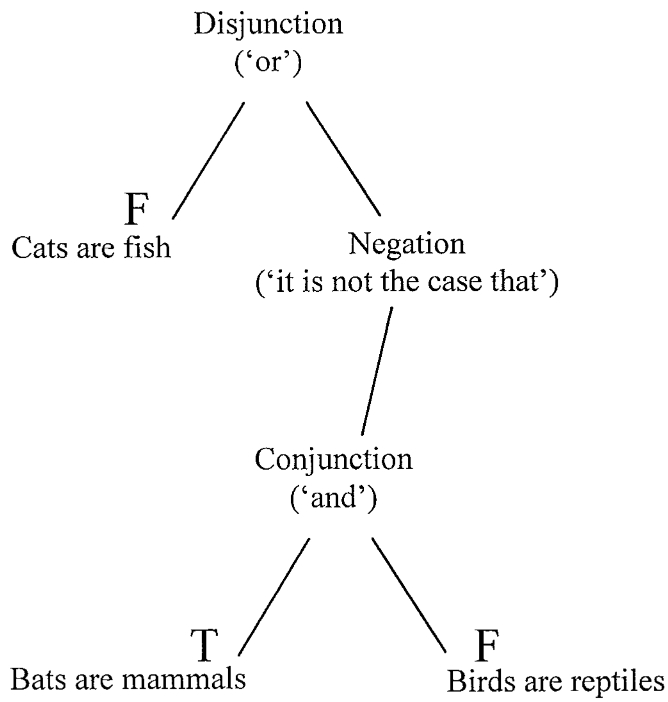
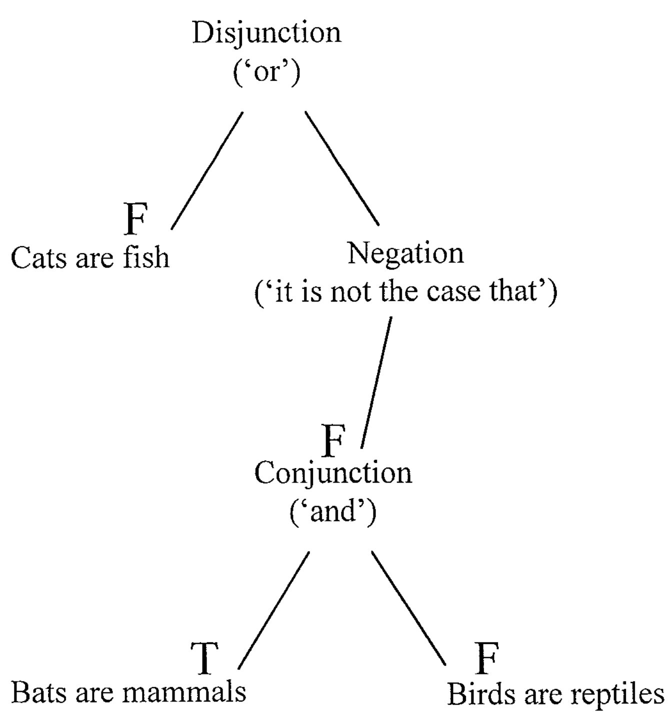


Next, we work our way up the tree, calculating the truth value of each logical operator and writing it in as we go. The first operator we come to is the conjunction. To determine its truth value, we check the truth values of its operands. One of them is true, and the other is false. What is the truth value of a conjunction with one true and one false conjunct? False. So we write that in above the operator:



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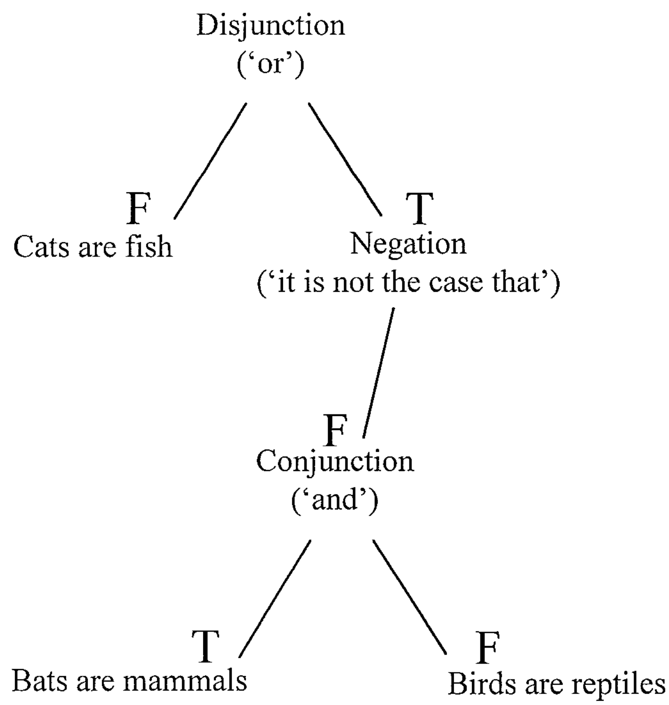
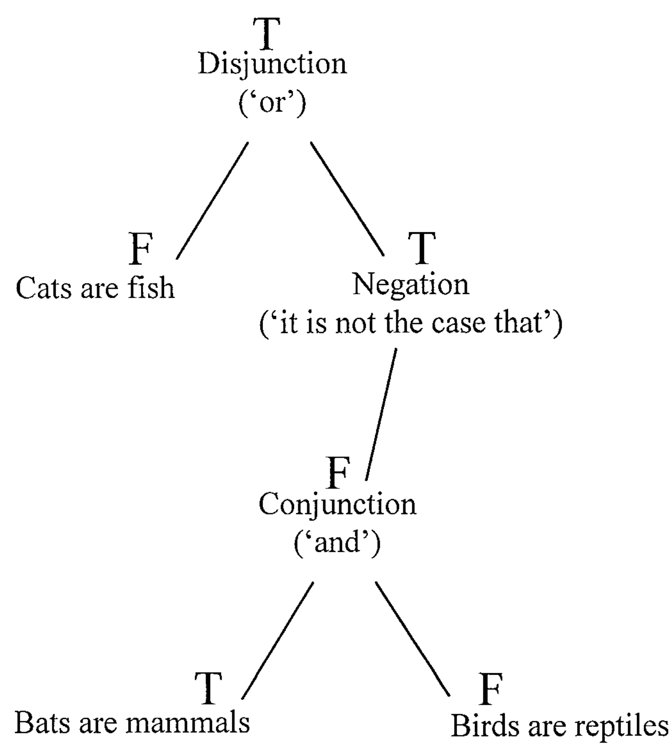
The next operator, as we work our way up, is the negation. Negation always reverses the truth value of its operand. In this case, the operand is false, so the negation will be true:



Finally, we have a disjunction with one true disjunct and one false disjunct. Such a disjunction (remember to consult the example Parsnip trees in the previous section) is always true, and so we label the operator at the top of our tree and we're done.



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What this exercise has told us is that statement (14) is true.
Let's try another one:

> It is not the case that St. Petersburg is not the capital of Russia and Moscow is not the capital of Russia. (15)

I would suggest that at this point you take a piece of scrap paper and draw the Parsnip tree for (15), and then evaluate it using the method we just worked through for (14). In fact, I'll give you another example to chew on before I reveal the solutions for both of them:

> Cadmium is an element, and it is not the case that barium is an element or that the symbol for tin is not 'Sn'. (16)

Work through both examples, and then compare what you come up with with the completed Parsnip trees on the next page.

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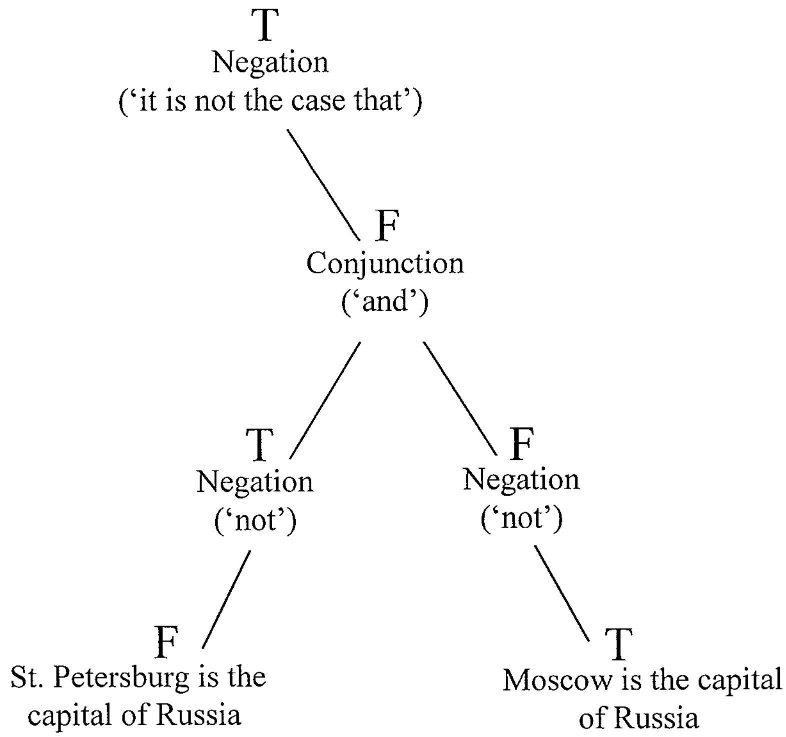
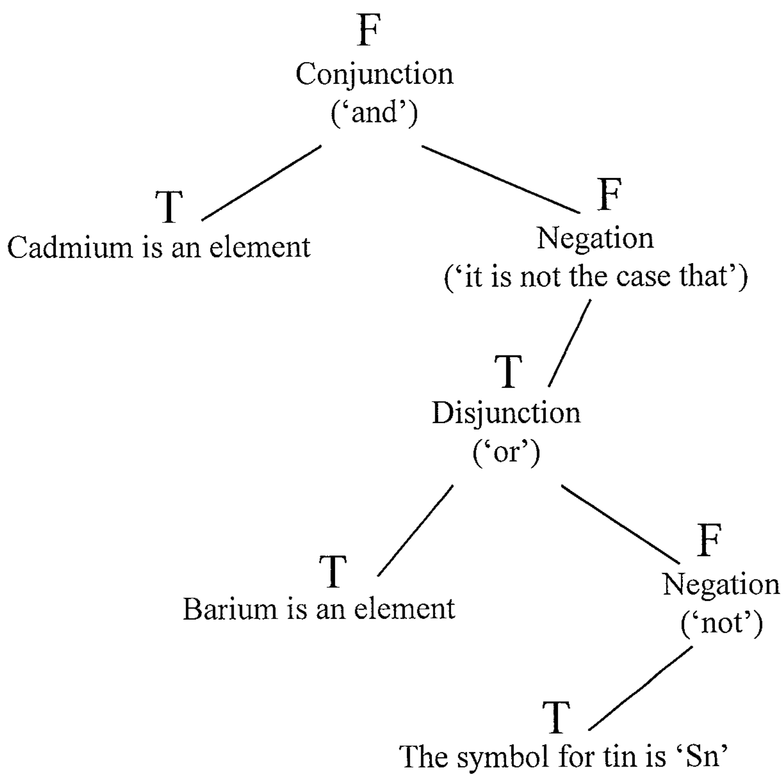
Here's the Parsnip tree, with evaluations, for (15):



How did you do on statement (15)? If your Parsnip tree and evaluation aren't just like mine, you should check yours and make sure you understand what's wrong with it.

Here's the solution for (16):



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Again, check your answer and make sure you understand each detail of the correct answer given above.

Terms and concepts discussed in this chapter

statement evaluation
using Parsnip trees for evaluation
truth value rule for negation
evaluation Parsnip trees for negation
truth value rule for conjunction
evaluation Parsnip trees for conjunction
truth value rule for disjunction
evaluation Parsnip trees for disjunction
general method for evaluating molecular statements using Parsnip trees

Exercises

Using the method set out in this chapter, evaluate each of the following molecular statements.

1. Albany is not the capital of New York State, or Miami is the capital of Florida.
2. It is not the case that New York isn't the capital of the U.S. and London is the capital of the U.K.
3. Munich is the capital of Germany or Berlin is the capital of Germany, and it is not the case that Paris is the capital of France and Lyons is the capital of France.
4. It isn't true that Milan is the capital of Italy or that it is not the case that Rome is the capital of Italy and Bern is the capital of Switzerland.
5. Geneva is the capital of Switzerland or Toronto is the capital of Canada, *or* Sydney is the capital of Australia or Shanghai isn't the capital of China.

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